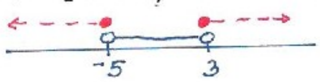
⑤ Urči doplněk množin v množině reálných čísel

a) $A = \{x \in R;\ x > 5\}$

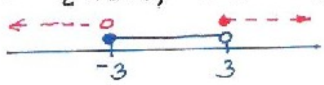
$A' = \{x \in R;\ x \leqq 5\} = (-\infty, 5\rangle$

b) $B = \{x \in R;\ -5 < x < 3\}$



$A' = \{x \in R;\ x \leqq -5 \wedge x \geqq 3\} = (-\infty, -5\rangle \underbrace{\cup}_{\text{sjednocení}} \langle 3, +\infty)$

c) $C = \{x \in R;\ -3 \leqq x < 3\}$



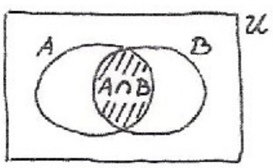
$A' = \{x \in R;\ x < -3 \wedge x \geqq 3\} = (-\infty, -3) \cup \langle 3, +\infty)$

- PRŮNIK MNOŽIN

Def. PRŮNIK MNOŽIN A, B (zn. $A \cap B$) je množina všech prvků, které patří zároveň do obou množin

[tj: patří do mn. A a současně do mn. B]

$$A \cap B = \{x \in \mathcal{U};\ x \in A \underset{\text{a současně}}{\wedge} x \in B\}$$

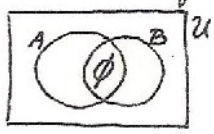


- VLASTNOSTI:

1. $A \cap \emptyset = \emptyset$ 2. $A \cap A = A$

- DISJUNKTNÍ MNOŽINY A, B: množiny, které nemají žádný společný prvek

- platí: $A \cap B = \emptyset$



Příklady

⑥ Urči průnik množin A, B

a) $A = \{1, 2, 5, 8\}$

$B = \{1, 3, 5, 7\}$

$A \cap B = \{1, 5\}$

b) $A = \{x \in N;\ x > 2\} = \{3, 4, 5, 6, 7, 8, 9, \dots\}$

$B = \{x \in N;\ x < 7\} = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{3, 4, 5, 6\} = \{x \in N;\ 3 \leqq x \leqq 6\}$
$= \{x \in N;\ 2 < x < 7\}$

c) $A = \{x \in Z;\ x > -3\} = \{-2, -1, 0, 1, \dots\}$

$B = \{x \in Z;\ x > -5\} = \{-4, -3, -2, -1, 0, 1, \dots\}$

$A \cap B = \{-2, -1, 0, 1, \dots\} = \{x \in Z;\ x > -3\}$
$= \{x \in Z;\ x \geqq -2\}$

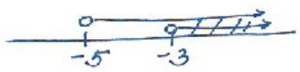
d) $A = N = \{1, 2, 3, 4, \dots\}$

$B = Z = \{\dots -2, -1, 0, 1, 2, 3, 4, \dots\}$

$A \cap B = \{1, 2, 3, 4, \dots\} = N$

e) $A = \{x \in R;\ x > -3\} = (-3, +\infty)$

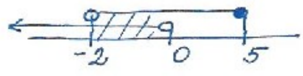
$B = \{x \in R;\ x > -5\} = (-5, +\infty)$



$A \cap B = (-3, +\infty) = \{x \in R;\ x > -3\}$

f) $A = \{x \in R;\ -2 < x \leqq 5\} = (-2, 5\rangle$

$B = \{x \in R;\ x < 0\} = (-\infty, 0)$

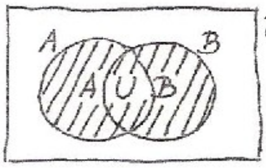


$A \cap B = (-2, 0)$
$= \{x \in R;\ -2 < x < 0\}$

DALŠÍ PŘÍKLADY – čl. INTERVALY (průnik)

– SJEDNOCENÍ MNOŽIN

Def. SJEDNOCENÍ MNOŽIN A, B (zn. $A \cup B$) je množina všech prvků, které patří alespoň do jedné z množin A, B
[tj. patří do mn. A nebo do mn. B]



$$A \cup B = \{x \in U;\ x \in A \underset{\text{nebo}}{\vee} x \in B\}$$

– VLASTNOSTI:

1. $A \cup A = A$
2. $A \cup \emptyset = A$
3. $A \cup A' = U$ (zákl. mn.)

Příklady

⑦ Urči sjednocení množin

a) $A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3, 5, 8\}$
$A \cup B = \{1, 2, 3, 5, 7, 8\}$
$[= \{x \in N;\ x \leq 8\} - \{4, 6\}]$

b) $A = \{x \in N;\ x > 3\} = \{4, 5, 6, 7, \ldots\}$
$B = \{x \in N;\ x < 8\} = \{1, 2, 3, 4, 5, 6, 7\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, \ldots\} = N$

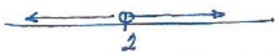
c) $A = N = \{1, 2, 3, \ldots\}$
$B = Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$A \cup B = B = Z$

d) $A = \{x \in Z;\ x > -5\} = \{-4, -3, -2, -1, 0, \ldots\}$
$B = \{x \in Z;\ x < -2\} = \{-3, -4, -5, \ldots\} = \{\ldots -5, -4, -3\}$
$A \cup B = \{\ldots -5, -4, -3, -2, -1, 0, \ldots\} = Z$

e) $A = \{x \in R;\ x > 2\}$
$B = \{x \in R;\ -2 \leq x < 7\}$

$A \cup B = \langle -2, +\infty)$

f) $A = \{x \in R;\ x > 2\} = (2, +\infty)$
$B = \{x \in R;\ x < 2\} = (-\infty, 2)$



$A \cup B = (-\infty, 2) \cup (2, +\infty) = R - \{2\}$

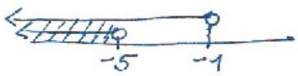
DALŠÍ PŘÍKLADY – uč. INTERVALY (sjednocení)

⑧ Urči průnik a sjednocení množin A, B

a) $A = \{-2, 0, 5, 7\}$, $B = \{-3, -1, 0, 4, 7, 9\}$
$A \cap B = \{0, 7\}$
$A \cup B = \{-3, -2, -1, 0, 4, 5, 7, 9\}$

b) $A = \{x \in Z;\ x < -5\} = \{-6, -7, -8, \ldots\}$
$B = \{x \in Z;\ x \leq -1\} = \{-1, -2, -3, -4, -5, -6, -7, -8, \ldots\}$
$A \cap B = \{-6, -7, -8, \ldots\} = \{x \in Z;\ x \leq -6\}$
$A \cup B = \{-1, -2, -3, -4, -5, -6, -7, \ldots\} = \{x \in Z;\ x \leq -1\}$

c) $A = \{x \in R;\ x < -5\} = (-\infty, -5)$
$B = \{x \in R;\ x \leq -1\} = (-\infty, -1\rangle$



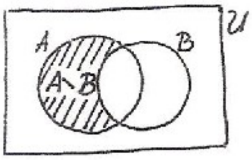
$A \cap B = (-\infty, -5)$

$A \cup B = (-\infty, -1)$

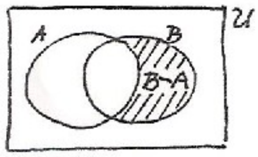
– ROZDÍL MNOŽIN

ROZDÍL MNOŽIN (mn. $A \setminus B$) je množina všech prvků množiny A, které nejsou prvky množiny B
[tj. patří do mn. A a nepatří do mn. B]

$A \setminus B = \{x \in \mathcal{U};\ x \in A \wedge x \notin B\}$ obdobně: $B \setminus A = \{x \in \mathcal{U};\ x \in B \wedge x \notin A\}$



OBECNĚ
$A \setminus B \neq B \setminus A$
(pro $A \neq B$)



Příklady

⑨ Urči rozdíl množin $A \setminus B$, $B \setminus A$

a) $A = \{1, 2, 5, 8\}$
$B = \{1, 3, 5, 7\}$
$A \setminus B = \{2, 8\}$
$B \setminus A = \{3, 7\}$

b) $A = \{x \in N;\ x > 2\} = \{3, 4, 5, 6, 7, 8, \dots\}$
$B = \{x \in N;\ x < 7\} = \{1, 2, 3, 4, 5, 6\}$
$A \setminus B = \{7, 8, 9, \dots\} = \{x \in N;\ x \geqq 7\}$
$B \setminus A = \{1, 2\}$

c) $A = \{x \in Z;\ x > -3\} = \{-2, -1, 0, 1, 2, \dots\}$
$B = \{x \in Z;\ x > -5\} = \{-4, -3, -2, -1, 0, 1, 2, \dots\}$
$A \setminus B = \emptyset$
$B \setminus A = \{-4, -3\}$

d) $A = N = \{1, 2, 3, 4, \dots\}$
$B = Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
$A \setminus B = \emptyset$
$B \setminus A = \{\dots -3, -2, -1, 0\} = Z_0^-$

3. Vennovy diagramy, slovní úlohy

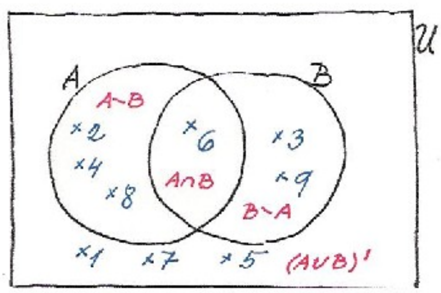
① Urči výčtem prvků množiny, piš i symbolickou definici a zakresli všechny prvky do Vennova diagramu:

[$2/x$ … 2 dělí x, tj. x je dělitelné

$\mathcal{U} = \{x \in N;\ x < 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{x \in \mathcal{U};\ 2/x\} = \{2, 4, 6, 8\}$
$B = \{x \in \mathcal{U};\ 3/x\} = \{3, 6, 9\}$

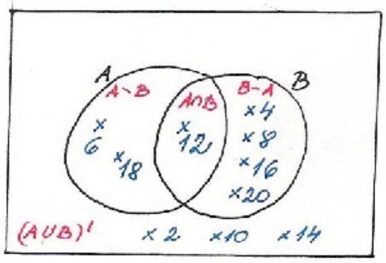
$A' = \{x \in \mathcal{U};\ x \notin A\} = \{1, 3, 5, 7, 9\}$
$B' = \{x \in \mathcal{U};\ x \notin B\} = \{1, 2, 4, 5, 7, 8\}$
$A \cap B = \{x \in \mathcal{U};\ x \in A \wedge x \in B\} = \{6\}$
$A \cup B = \{x \in \mathcal{U};\ x \in A \vee x \in B\} = \{2, 3, 4, 6, 8, 9\}$
$A \setminus B = \{x \in \mathcal{U};\ x \in A \wedge x \notin B\} = \{2, 4, 8\}$
$B \setminus A = \{x \in \mathcal{U};\ x \in B \wedge x \notin A\} = \{3, 9\}$

$(A \cup B)' = \{x \in \mathcal{U};\ x \notin A \cup B\} = \{1, 5, 7\}$



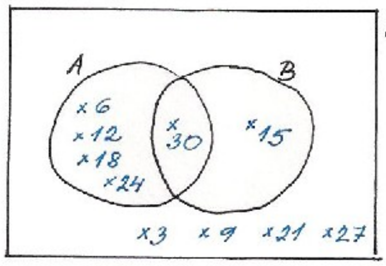
b) $\mathcal{U} = \{x \in N;\ 2/x \wedge x \leqq 20\} = \{2,4,6,8,10,12,14,16,18,20\}$
DÚ
$A = \{x \in \mathcal{U};\ 3/x\} = \{6,12,18\}$
$B = \{x \in \mathcal{U};\ 4/x\} = \{4,8,12,16,20\}$

$A' = \{x \in \mathcal{U};\ x \notin A\} = \{2,4,8,10,14,16,20\}$
$B' = \{x \in \mathcal{U};\ x \notin B\} = \{2,6,10,14,18\}$
$A \cap B = \{x \in \mathcal{U};\ x \in A \wedge x \in B\} = \{12\}$
$A \cup B = \{x \in \mathcal{U};\ x \in A \vee x \in B\} = \{4,6,8,12,16,18,20\}$
$A \setminus B = \{x \in \mathcal{U};\ x \in A \wedge x \notin B\} = \{6,18\}$
$B \setminus A = \{x \in \mathcal{U};\ x \in B \wedge x \notin A\} = \{4,8,16,20\}$
$(A \cup B)' = \{x \in \mathcal{U};\ x \notin A \cup B\} = \{2,10,14\}$



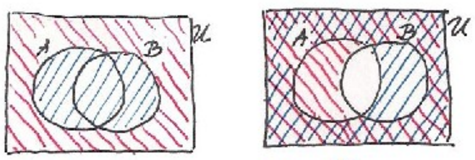
c) $\mathcal{U} = \{x \in N;\ 3/x \wedge x \leqq 30\} = \{3,6,9,12,15,18,21,24,27,30\}$
DÚ
$A = \{x \in \mathcal{U};\ 2/x\} = \{6,12,18,24,30\}$
$B = \{x \in \mathcal{U};\ 5/x\} = \{15,30\}$

$A' = \{x \in \mathcal{U};\ x \notin A\} = \{3,9,15,21,27\}$
$B' = \{x \in \mathcal{U};\ x \notin B\} = \{3,6,9,12,18,21,24,27\}$
$A \cap B = \{x \in \mathcal{U};\ x \in A \wedge x \in B\} = \{30\}$
$A \cup B = \{x \in \mathcal{U};\ x \in A \vee x \in B\} = \{6,12,15,18,24,30\}$
$A \setminus B = \{x \in \mathcal{U};\ x \in A \wedge x \notin B\} = \{6,12,18,24\}$
$B \setminus A = \{x \in \mathcal{U};\ x \in B \wedge x \notin A\} = \{15\}$
$(A \cup B)' = \{x \in \mathcal{U};\ x \notin (A \cup B)\} = \{3,9,21,27\}$



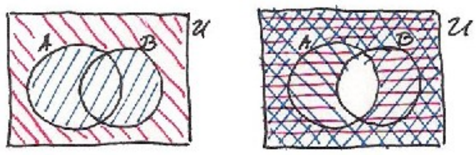
② Užitím Vennova diagramu zjistěte, zda platí:

a) $(A \cup B)' = A' \cap B'$



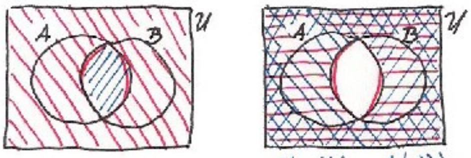
$A \cup B$ ///
$(A \cup B)'$ \\\
A' ///
B' \\\
$A' \cap B'$ XXX
POROVNÁME => PLATÍ

b) $(A \cup B)' = A' \cup B'$



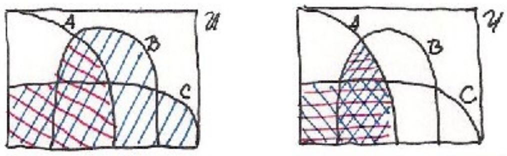
$A \cup B$ ///
$(A \cup B)'$ \\\
A' ///
B' \\\
NEPLATÍ $A' \cup B'$ ≡

c) $(A \cap B)' = A' \cup B'$



$A \cap B$ ///
$(A \cap B)'$ \\\
PLATÍ
A' /// B' \\\
$A' \cup B'$ ≡

d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



$B \cup C$ ///
$A \cap (B \cup C)$ \\\
PLATÍ
$A \cap B$ /// $A \cap C$ \\\
$(A \cap B) \cup (A \cap C)$ ≡

e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
DÚ [platí]

f) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
DÚ [platí]