8. Rovnice a nerovnice s absolutními hodnotami (i kvadratické)

$$|a| = \begin{cases} a & \text{pro } a \geq 0 \\ -a & \text{pro } a < 0 \end{cases}$$

$|a \cdot b| = |a| \cdot |b|$

[! NELZE $|a+b| = |a| + |b|$]

Příklady

① Řešte v $\mathbb{R}$

a) $|2x-1| = x^2 - 2$

(1. způsob) na základě def. abs. hodnoty (vhodné pro 1 abs. h.)

1. $2x-1 \geq 0 \ \dots \ |2x-1| = 2x-1$ a řeším dál: $2x-1 = x^2-2$

$2x \geq 1$

$x \geq \frac{1}{2}$

$D_1 = \langle \frac{1}{2}, +\infty)$

$x^2 - 2x - 1 = 0$

$(a=1, b=-2, c=-1)$

$x_{1,2} = \frac{2 \pm \sqrt{4+4 \cdot 1 \cdot 1}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = \frac{2(1 \pm \sqrt{2})}{2}$

$x_{1,2} = 1 \pm \sqrt{2}$ $x_1 = 1+\sqrt{2} \doteq 2{,}4 \in D$ $x_2 = 1-\sqrt{2} \notin D$

$K_1 = \{1+\sqrt{2}\}$

2. $2x-1 < 0 \ \dots \ |2x-1| = 1-2x \ \dots \ 1-2x = x^2-2$

$2x < 1$

$x < \frac{1}{2}$

$D_2 = (-\infty, \frac{1}{2})$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$x_1 = -3 \in D_2$ $x_2 = 1 \notin D_2$

$K_2 = \{-3\}$

$K = K_1 \cup K_2 = \{-3;\ 1+\sqrt{2}\}$

(2. způsob) metoda nul. b. (tabulka na odstr. abs. h.)

$|2x-1| = x^2 - 2$

[nul. b. $2x-1=0$, $2x=1$, $x = \frac{1}{2}$]

	D_1: $(-\infty, \frac{1}{2})$	D_2: $\langle \frac{1}{2}, +\infty)$
$\|2x-1\|$	⊖ $1-2x$	⊕ $2x-1$

např. 1: $2 \cdot 1 - 1 > 0$

1. $D_1 = (-\infty, \frac{1}{2}) \ \dots \ 1-2x = x^2-2$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x_1 = -3 \in D_1$ $x_2 = 1 \notin D_1$

$K_1 = \{-3\}$

2. $D_2 = \langle \frac{1}{2}, +\infty) \ \dots \ 2x-1 = x^2-2$

$0 = x^2 - 2x - 1$

$x_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$

$x_{1,2} = \frac{2(1 \pm \sqrt{2})}{2} = 1 \pm \sqrt{2}$

$x_1 = 1+\sqrt{2} \in D_2$ $x_2 = 1-\sqrt{2} \notin D_2$

$K_2 = \{1+\sqrt{2}\}$

$K = K_1 \cup K_2$

$K = \{-3,\ 1+\sqrt{2}\}$

b) $|x^2+3x|-4=0$

$|x(x+3)|-4=0$

$|x|\cdot|x+3|-4=0$

[nul. b. $0, -3$

$-3 \quad 0$]

$|a\cdot b| = |a|\cdot|b|$

	D_1: $(-\infty,-3)$	$\langle -3,0)$	$\langle 0,+\infty)$	
$\lvert x\rvert$	⊖ $-x$	⊖ $-x$	⊕ x	nr. 1: $1>0$
$\lvert x+3\rvert$	⊖ $-(x+3) = -x-3$	⊕ $x+3$	⊕ $x+3$	nr. 1: $1+3>0$

1. $D_1=(-\infty,-3)$ … $-x(-x-3)-4=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x_1=-4 \in D_1$ $\quad$ $x_2=1 \notin D_1$

$\mathcal{K}_1=\{-4\}$

2. $x\in\langle-3,0)$ … $-x(x+3)-4=0$

$-x^2-3x-4=0$

$x^2+3x+4=0$

$x_{1,2}=\dfrac{-3\pm\sqrt{9-4\cdot4}}{2}$, $D<0$

$\mathcal{K}_2=\emptyset$

3. $D_3=\langle0,+\infty)$ … $x(x+3)-4=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x_1=-4 \notin D_3$ $\quad$ $x_2=1 \in D_3$

$\mathcal{K}_2=\{1\}$

$\mathcal{K}=\mathcal{K}_1\cup\mathcal{K}_2\cup\mathcal{K}_3$

$\mathcal{K}=\{-4,1\}$

(2. způ.) – s využ. kvadr. nerov

$|x^2+3x|-4=0$

$|\underbrace{x(x+3)}_{}|-4=0$ $\quad$ (a)

$a\geq0$

1. $x(x+3)\geq0$ … $x^2+3x-4=0$

$(x+4)(x-1)=0$

$x_1=-4 \in D_1$ $\quad$ $x_2=1 \in D_1$

$D_1=(-\infty,0\rangle\cup\langle-3,+\infty)$

$a<0$

2. $x(x+3)<0$ … $-x^2-3x-4=0$ $/\cdot(-1)$

$x^2+3x+4=0$

$x_{1,2}=\dfrac{-3\pm\sqrt{9-16}}{2}$, $D<0$

nemá v R řeš.

$\mathcal{K}_2=\emptyset$

$\mathcal{K}=\mathcal{K}_1\cup\mathcal{K}_2=\{-4,1\}$

c) $|x^2+3x+4|=x+3$

[nul. b. $x^2+3x+4=0$

$x_{1,2}=\dfrac{-3\pm\sqrt{9-4\cdot4}}{2}$, $D<0$

nemá nul. b.]

$x^2+3x+4>0$ pro $\forall x\in R$

$\Rightarrow |x^2+3x+4|=x^2+3x+4$

$\Rightarrow$ rce má tvar … $x^2+3x+4=x+3$

$x^2+2x+1=0$

$(x+1)(x+1)=0$

$(x+1)^2=0$

$x_{1,2}=-1\in R$

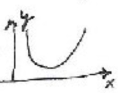
$\mathcal{K}=\{-1\}$

d) $|x^2+3x+4|=x$

[nul. b. $x_{1,2}=\dfrac{-3\pm\sqrt{9-4\cdot1\cdot4}}{2}$, $D<0$]

nul. b. nem. $\Rightarrow$

$x^2+3x+4>0$ pro $\forall x\in R$



$\Rightarrow$ rce má tvar $x^2+3x+4=x$

$x^2+2x+4=0$

$x_{1,2}=\dfrac{-2\pm\sqrt{4-16}}{2}$, $D<0$

$\mathcal{K}=\emptyset$

③ řešte v R

a) $x^2 > x|x| + x$ $\qquad \mathcal{D} = R$

1. $x \geq 0 \ldots$ $x^2 > x^2 + x$

$0 > x$

$x < 0$

$\mathcal{D}_1 = \langle 0, +\infty)$ řešíme v $\mathcal{D}_1$ => průnik

$\mathcal{K}_1 = \emptyset$

2. $x < 0 \ldots$ $x^2 > x \cdot (-x) + x$ $\quad |x| = -x$

$\mathcal{D}_2 = (-\infty, 0)$

$x^2 > -x^2 + x$ kr. nerov. – ANULOVAT

$2x^2 - x > 0$ na souč. tvar

$x(2x-1) > 0$

$[x_0 = 0,\ 2x - 1 = 0,\ 2x = 1,\ x_0 = \frac{1}{2}]$

nebo $+ \ 0 \ - \ \frac{1}{2} \ +$ pr. 1 $1(2-1)$ ⊕

$x \in (-\infty, 0) \cup (\frac{1}{2}, +\infty)$

průnik

$\mathcal{K}_2 = (-\infty, 0)$

sjednocení

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2$

$\mathcal{K} = \emptyset \cup (-\infty, 0)$

$\mathcal{K} = (-\infty, 0) = R^-$

b) $|x^2 + 2x - 3| > 2x + 1$ $\quad (a = x^2+2x-3)$

1. $x^2 + 2x - 3 \geq 0$ ($a \geq$) $\ldots$ $x^2 + 2x - 3 > 2x + 1$ ($|a| = a$)

$(x+3)(x-1) \geq 0$ $\qquad x^2 - 4 > 0$

$[x_0 = -3,\ x_0 = 1]$ $\qquad (x-2)(x+2) > 0$

$[x_0 = 2,\ x_0 = -2]$

$\mathcal{D}_1 = (-\infty, -3\rangle \cup \langle 1, +\infty)$

$x \in (-\infty, -2) \cup (2, +\infty)$

(zjistit, která x patří do $\mathcal{D}_2$ – průnik)

$\mathcal{K}_1 = (-\infty, -3\rangle \cup (2, +\infty)$

2. $x^2 + 2x - 3 < 0$ ($a < 0$) $\ldots$ $-(x^2 + 2x - 3) > 2x + 1$ ($|a| = -a$)

$(x+3)(x-1) < 0$ $\qquad -x^2 - 2x + 3 > 2x + 1$

$[x_0 = -3,\ x_0 = 1]$ $\qquad -x^2 - 4x + 2 > 0 \quad /\cdot(-1)$

$x^2 + 4x - 2 < 0$

$\mathcal{D}_2 = (-3, 1)$

nul. b. $x_{1/2} = \dfrac{-4 \pm \sqrt{16 + 4 \cdot 2}}{2} = \dfrac{-4 \pm \sqrt{24}}{2} = \dfrac{-4 \pm 2\sqrt{6}}{2} = \dfrac{2(-2 \pm \sqrt{6})}{2}$

$x_{1/2} = -2 \pm \sqrt{6}$

$x \in (-2-\sqrt{6}, -2+\sqrt{6})$ $\quad \underbrace{2 < 3}_{< 1}$

$\mathcal{K}_2 = (-3, -2+\sqrt{6})$

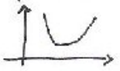
$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = (-\infty, -3\rangle \cup (2, +\infty) \cup (-3, -2+\sqrt{6}) = (-\infty, -2+\sqrt{6}) \cup (2, +\infty)$

c) $|x^2-2x+5| \geq 2x^2-3$ $\quad D=R$

1. $x^2-2x+5 \geq 0$

[nul. b. $x_{1,2}=\frac{2\pm\sqrt{4-20}}{2}$, $D<0$

=> nem. nul. b.



=> $a=1>0 \dots x^2-2x+5>0$]

=> $|x^2-2x+5| = x^2-2x+5$ pro $\forall x \in R$

nerov. má tvar

$x^2-2x+5 \geq 2x^2-3$

$-x^2-2x+8 \geq 0 \quad | \cdot(-1)$

$x^2+2x-8 \overset{!}{\leq} 0$

$(x+4)(x-2) \leq 0$

[nul. b. $-4, 2$]

$x \in \langle -4, 2 \rangle$

$D_1 = R$

$K = \langle -4, 2 \rangle$

[2. $x^2-2x+5<0$
pro žádné $x \in R$ => $D_2=\emptyset$ a nemá smysl nerov. řešit]

③ Řešte v R

DÚ a) $|4x-1| = x^2+2x+3$

1. $4x-1 \geq 0 \quad \dots \quad 4x-1 = x^2+2x+3$

$4x \geq 1 \qquad 0 = x^2-2x+4$

$x \geq \frac{1}{4} \qquad x_{1,2}=\frac{2\pm\sqrt{4-4\cdot4}}{2}$, $D<0$

$D_1 = \langle \frac{1}{4}, +\infty)$ $\qquad$ nemá řeš.

$K_1 = \emptyset$

2. $4x-1<0 \quad \dots \quad -4x+1 = x^2+2x+3$

$x<\frac{1}{4} \qquad 0 = x^2+6x+2$

$D_2 = (-\infty, \frac{1}{4})$

$x_{1,2}=\frac{-6\pm\sqrt{36-4\cdot2}}{2}$

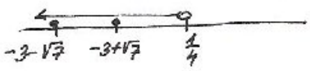
$x_{1,2}=\frac{-6\pm\sqrt{28}}{2}$

$x_{1,2}=\frac{-6\pm2\sqrt{7}}{2}$

průnik

$x_{1,2}=-3\pm\sqrt{7}$

$x_1=-3+\sqrt{7} \doteq -0{,}35$



$x_2=-3-\sqrt{7} \doteq -5{,}65$

$K_2 = \{-3-\sqrt{7}, -3+\sqrt{7}\}$

$K = K_1 \cup K_2$

$K = \emptyset \cup \{-3-\sqrt{7}, -3+\sqrt{7}\}$

$K = \{-3-\sqrt{7}, -3+\sqrt{7}\}$

b) $|2-y| = \frac{1}{2}y^2-y+3$

1. $2-y \geq 0 \quad \dots \quad 2-y = \frac{1}{2}y^2-y+3$

$-y \geq -2 \qquad 4-2y = y^2-2y+6$

$y \overset{!}{\leq} 2 \qquad 0 = y^2+2$

$D_1 = (-\infty, 2\rangle \qquad y^2=-2$ nelze v R

$K_1 = \emptyset$

2. $2-y<0 \quad \dots \quad -2+y = \frac{1}{2}y^2-y+3$

$-y<-2 \qquad -4+2y = y^2-2y+6$

$y \overset{!}{>} 2 \qquad 0 = y^2-4y+10$

$y_{1,2}=\frac{4\pm\sqrt{16-4\cdot10}}{2}$, $D<0$

$K_2 = \emptyset$

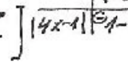
$K = K_1 \cup K_2$

$K = \emptyset$

[PŘÍKLADY LZE ŘEŠIT I TABULKOU JAKO PRO VÍCE ABS. H.

$|4x-1| = x^2+2x+3$

[nul. b. $x=\frac{1}{4}$



]

	$(-\infty, \frac{1}{4})$	$\langle \frac{1}{4}, +\infty)$
$\lvert 4x-1 \rvert$	⊖ $1-4x$	⊕ $4x-1$

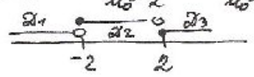
nr. 1

1. $D_1 = (-\infty, \frac{1}{4}) \dots 1-4x = x^2+2x+3$
⋮ dále viz výše 2.

2. $D_2 = \langle \frac{1}{4}, +\infty) \dots$ dále viz výše 1.]

c) $|u+2| + |u-2| = u^2+4$

[nul. b. $u+2=0$ $u_0=-2$ $u-2=0$ $u_0=2$



]

2 abs. h. => TABULKA NA ODSTRANĚNÍ ABS. H.

	$\mathcal{D}_1$ $(-\infty,-2)$	$\mathcal{D}_2$ $\langle -2, 2)$	$\mathcal{D}_3$ $\langle 2,+\infty)$	
$u+2$	⊖ $-u-2$	⊕ $u+2$	⊕ $u+2$	nr. 3: 3+2 ⊕
$u-2$	⊖ $2-u$	⊖ $2-u$	⊕ $u-2$	nr. 3: 3-2 ⊕

1. $\mathcal{D}_1 = (-\infty,-2)$ ... $-u-2+2-u = u^2+4$
[nebo $x \in (-\infty,-2)$]

$0 = u^2+2u+4$

$u_{1/2} = \dfrac{-2 \pm \sqrt{4-16}}{2}$, $D<0$

$\mathcal{K}_1 = \emptyset$

2. $\mathcal{D}_2 = \langle -2, 2)$ ... $u+2+2-u = u^2+4$

$0 = u^2$

$u = 0 \in \mathcal{D}_2$

$\mathcal{K}_2 = \{0\}$

3. $\mathcal{D}_3 = \langle 2,+\infty)$ ... $u+2+u-2 = u^2+4$

$2u = u^2+4$

$u^2-2u+4=0$

$u_{1/2} = \dfrac{2 \pm \sqrt{4-16}}{2}$, $D<0$

$\mathcal{K}_3 = \emptyset$

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 \cup \mathcal{K}_3$

$\mathcal{K} = \emptyset \cup \{0\} \cup \emptyset$

$\mathcal{K} = \{0\}$

d) $|y^2-4| = 3y^2$

1. $y^2-4 \geqq 0$ ... $y^2-4 = 3y^2$

$(y-2)(y+2) \geqq 0$ $0 = 2y^2+4$

[nul. b. $2, -2$] $y^2 = -2$ nelze v R

$\mathcal{D}_1 = (-\infty,-2\rangle \cup \langle 2,+\infty)$ $\mathcal{K}_1 = \emptyset$

2. $y^2-4<0$ ... $-(y^2-4) = 3y^2$

$(y-2)(y+2)<0$ $-y^2+4 = 3y^2$

nul. b. $2, -2$ $4 = 4y^2$

$y^2 = 1$

$|y| = 1$

$\mathcal{D}_2 = (-2, 2)$ $y_{1/2} = \pm 1 \in \mathcal{D}_2$

$\mathcal{K}_2 = \{1,-1\}$

[nebo $y^2-1=0$
$(y-1)(y+1)=0$
$y_1 = 1$, $y_2 = -1$]

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \emptyset \cup \{1,-1\} = \{1,-1\}$

e) $|1-2x+x^2| + x-1=0$

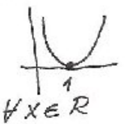
$|x^2-2x+1| + x-1=0$

1. $x^2-2x+1 \geqq 0$ ... $x^2-2x+1+x-1=0$

$(x-1)(x-1) \geqq 0$ $x^2-x = 0$

$(x-1)^2 \geqq 0$ $x(x-1)=0$

$x_1 = 0 \in \mathcal{D}_1$ $x_2 = 1 \in \mathcal{D}_1$



pl. pro $\forall x \in R$

$\mathcal{D}_1 = R$

$\mathcal{K} = \{0, 1\}$

[2. je-li $\mathcal{D}_1 = R$ $\Rightarrow \mathcal{D}_2 = \emptyset$, tj. $x^2-2x+1<0$ pro žádné $x \in R$ $\Rightarrow$ ani nemá smysl počítat]

NEŘEŠENÉ

f) $|x^2-2x+2| = 17$

[$\mathcal{K} = \{5,-3\}$ obd. jako g)]

g) $4 + |2x^2-x-6| = 0$

[$\mathcal{K} = \emptyset$, kořeny nepatří do $\mathcal{D}_1, \mathcal{D}_2$]

h) $|v| + v^2 = |2v-1|$

[tabulka $\mathcal{K} = \left\{ \dfrac{-1-\sqrt{5}}{2}, \dfrac{-3+\sqrt{13}}{2} \right\}$]

④ ŘEŠTE V $\mathbb{R}$ STAČÍ ŘEŠIT 1 ZPŮSOBEM

DÚ a) $|x-1| \leqq x^2-1$

1. zp. – z def. abs. h.

1. $x-1 \geqq 0 \quad \dots \quad x-1 \leqq x^2-1$

$x \geqq 1 \qquad 0 \leqq x^2-x$

$D_1 = \langle 1,+\infty)$

$x^2-x \geqq 0$

$x(x-1) \geqq 0$

[nul.b. 0, 1]

[str. 2 2(2-1)]

$x \in (-\infty,0\rangle \cup \langle 1,+\infty)$

(zjistit, zda patří do D_1)

průnik

$\mathcal{K}_1 = \langle 1,+\infty)$

2. $x-1<0 \quad \dots \quad 1-x \leqq x^2-1$

$x<1 \qquad 0 \leqq x^2+x-2$

$D_2 = (-\infty,1)$

$x^2+x-2 \geqq 0$

$(x+2)(x-1) \geqq 0$

[nul.b. −2, 1]

[str. 2 ⊕]

$x \in (-\infty,-2\rangle \cup \langle 1,+\infty)$

$\mathcal{K}_2 = (-\infty,-2\rangle \cup (1,+\infty)$

sjednocení

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \langle 1,+\infty) \cup (-\infty,-2\rangle \cup (1,+\infty)$

$\mathcal{K} = (-\infty,-2\rangle \cup \langle 1,+\infty)$

2. zp. – nul.b. a tab.

$|x-1| \leqq x^2-1$

[nul.b. $x-1=0$, $x=1$]

	D_1 $(-\infty,1)$	D_2 $\langle 1,+\infty)$
$\lvert x-1\rvert$	⊖ $1-x$	0 ⊕ $x-1$

1. $D_1 = (-\infty,1) \dots 1-x \leqq x^2-1$

$0 \leqq x^2+x-2$

dále viz 1. zp. – 2. část

2. $D_2 = \langle 1,+\infty) \dots x-1 \leqq x^2-1$

$0 \leqq x^2-x$

dále viz 1. zp. – 1. část

b) $|x^2-2x| < x$

1. zp. – na zákl. def. abs. h.

1. $x^2-2x \geqq 0$ ($a \geqq a$) $|a|=a$ $\qquad x^2-2x < x$

$x(x-2) \geqq 0 \qquad x^2-3x<0$

[nul.b. 0, 2] $\qquad x(x-3)<0$

[nul.b. 0, 3]

$D_1 = (-\infty,0) \cup \langle 2,+\infty)$

$x \in (0,3)$

průnik

$\mathcal{K}_1 = \langle 2,3)$

2. $x^2-2x<0$ ($a<0$) $|a|=-a$ $\quad \dots \quad 2x-x^2<x$

$x(x-2)<0 \qquad 0<x^2-x$

$D_2 = (0,2)$

$x^2-x>0$

$x(x-1)>0$

[nul.b. 0, 1]

$x \in (-\infty,0) \cup (1,+\infty)$

průnik

$\mathcal{K}_2 = (1,2)$

sjednocení

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \langle 2,3) \cup (1,2)$

$\mathcal{K} = (1,3)$

2. zp. – na 2 abs. h. + tab. (delší)

$|x^2-2x|<x$

$|x(x-2)|<x$

$|x|\cdot|x-2|<x$

[nul.b. 0, 2]

	D_1 $(-\infty,0)$	D_2 $\langle 0,2)$	D_3 $\langle 2,+\infty)$
$\lvert x\rvert$	⊖ $-x$	0 ⊕ x	⊕ x
$\lvert x-2\rvert$	$2-x$	$2-x$	0 ⊕ $x-2$

str. 3

1. $\mathcal{D}_1 = (-\infty, 0)$

$-x(2-x) < x$

$-2x + x^2 < x$

$x^2 - 3x < 0$

$x(x-3) < 0$

[nul.b. 0, 3]

$x \in (0, 3)$

$\underset{\sim\sim}{\mathcal{K}_1 = \emptyset}$

2. $\mathcal{D}_2 = \langle 0, 2)$

$x(2-x) < x$

$2x - x^2 < x$

$0 < x^2 - x$

$x^2 - x > 0$

$x(x-1) > 0$

[nul.b. 0, 1]

$x \in (-\infty, 0) \cup (1, +\infty)$

$\underset{\sim\sim}{\mathcal{K}_2 = (1, 2)}$

3. $\mathcal{D}_3 = \langle 2, +\infty)$

$x(x-2) < x$

$x^2 - 2x < x$

$x^2 - 3x < 0$ (viz 1.)

$x \in (0, 3)$

$\underset{\sim\sim}{\mathcal{K}_3 = \langle 2, 3)}$

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 \cup \mathcal{K}_3$

$\mathcal{K} = \emptyset \cup (1, 2) \cup \langle 2, 3)$

$\underset{\sim\sim}{\mathcal{K} = (1, 3)}$

c) $|1 - x^2| \leq \frac{1}{2}(x+1)$

1. $1 - x^2 \geq 0$ . . . $1 - x^2 \leq \frac{1}{2}(x+1)$ | $\cdot 2$

$(1-x)(1+x) \geq 0$

[nul.b. 1, −1]

nr. 2 $(1-2)(1+2)$ $\ominus$

$\mathcal{D}_1 = \langle -1, 1 \rangle$

$2 - 2x^2 \leq x + 1$

$0 \leq 2x^2 + x - 1$

$$\left[\begin{array}{l} \text{nul.b.} \\ x_{1,2} = \frac{-1 \pm \sqrt{1 + 4 \cdot 2 \cdot 1}}{4} \\ x_{1,2} = \frac{-1 \pm 3}{4} = \begin{cases} \frac{-1+3}{4} = \frac{1}{2} \\ -\frac{4}{4} = -1 \end{cases} \end{array}\right]$$

$x \in (-\infty, -1\rangle \cup \langle \frac{1}{2}, +\infty)$

!! koeficient u $x^2 = -1 < 0$ => PARABOLA rozevřená dolů

průnik

$\underset{\sim\sim}{\mathcal{K}_1 = \{-1\} \cup \langle \frac{1}{2}, 1 \rangle}$

2. $1 - x^2 < 0$ . . . $x^2 - 1 \leq \frac{1}{2}(x+1)$

$(1-x)(1+x) < 0$

[nul.b. 1, −1]

$\mathcal{D}_2 = (-\infty, -1) \cup (1, +\infty)$

$2x^2 - 2 \leq x + 1$

$2x^2 - x - 3 \leq 0$

$$\left[\begin{array}{l} \text{nul.b.} \\ x_{1,2} = \frac{1 \pm \sqrt{1 + 4 \cdot 2 \cdot 3}}{4} \\ x_{1,2} = \frac{1 \pm 5}{4} = \begin{cases} \frac{3}{2} \\ -1 \end{cases} \end{array}\right]$$

$x \in \langle -1, \frac{3}{2} \rangle$

průnik

$\underset{\sim\sim}{\mathcal{K}_2 = (1, \frac{3}{2} \rangle}$

sjednocení

$\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2 = \{-1\} \cup \langle \frac{1}{2}, 1 \rangle \cup (1, \frac{3}{2} \rangle$

$\underset{\sim\sim}{\mathcal{K} = \{-1\} \cup \langle \frac{1}{2}, \frac{3}{2} \rangle}$

2.zp. – na 2 abs.h.

$|1 - x^2| = |(1-x) \cdot (1+x)| = |1-x||1+x|$

=> nul.b., tab., $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$ atd.

NEŘEŠENÉ

d) $|x^2 - 2x - 3| < 3x - 3$

[$\mathcal{K} = (2, 5)$]

e) $|x^2 - 4x| < 5$

[$\mathcal{K} = (-1, 5)$]

f) $|x - 6| < x^2 - 5x + 9$

[$\mathcal{K} = (-\infty, 1) \cup (3, +\infty)$]